Principles of Analysis (Math 3083)
Final Study Guide
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Advice for the Final

(a) Start at the beginning of the course and review all of the material. Be familiar with all of the major definitions and concepts. Pay particular attention to induction, sequences in $\mathbb{R}$, and various metric space ideas such as bounded sets, open sets, closed sets, compactness, connectedness, and completeness. Print Problem Set E and do as many of the problems as you can.

(b) Be sure that you are comfortable with the solutions to all of the previous exams, quizzes, and problem sets.

(c) Print and review the metric space notes from the web site.

(d) Print Problem Set E and do as many of these problems as you can. This problem set is due at the time of the final, and will count for extra credit.

Errata

Problems 2 and 3 from Problem Set E handed out in class need to be modified. Reprint Problem Set E from the web site.

Preview

Material on the final will include the following metric spaces:

- $\mathbb{R}$ is the set of real numbers, together with its standard metric
$$\rho(x, y) = |x - y|.$$
- $\mathbb{Q}$ is the set of rational numbers, viewed as a subspace of $\mathbb{R}$.
- $\mathbb{K} = [0, 1] \subset \mathbb{R}$ is the closed unit interval, viewed as a subspace of $\mathbb{R}$.
- $\mathbb{F}$ is the set of all continuous functions $f : \mathbb{K} \to \mathbb{K}$, together with the metric
$$\rho(f, g) = \max\{|f(x) - g(x)| : x \in \mathbb{K}\}.$$
- $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$, together with its standard metric
$$\rho(p, q) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2},$$
where $p = (x_1, y_1)$ and $q = (x_2, y_2)$.

Select various subsets of these spaces and decide if they are bounded, open, closed, compact, connected, or complete. Find their closure, interior, and boundary.